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EARLY ALGEBRA IN ELEMENTARY SCHOOLS: TRENDS, APPROACHES, AND CHALLENGES

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Abstract

Early algebra constitutes an essential foundation in elementary education because it supports students' relational reasoning, structural understanding, and generalization abilities from an early age. However, many studies indicate that algebra instruction in elementary schools remains dominated by procedural arithmetic approaches, which often lead to persistent conceptual difficulties and misconceptions, particularly during the transition to formal algebra. Therefore, this study aims to review and synthesize research on early algebra in elementary education to provide a systematic overview of research foci, instructional approaches, and existing limitations and research gaps. This study employed a literature review of articles retrieved from Google Scholar using the keywords early algebra, early algebra in elementary schools, and early algebra in primary education published between 2020 and 2025. A total of 49 articles were collected and screened based on relevance criteria, resulting in 39 articles for analysis. The research instrument was an article analysis sheet, and the data were analyzed using thematic analysis by classifying the articles into eight categories of study. The results show that early algebra research primarily focuses on identifying students' difficulties and misconceptions, particularly related to variables, the meaning of the equal sign, and the arithmetic-to-algebra transition. Dominant instructional approaches include multiple representations, pattern- and relationship-based tasks, problem-based learning, and game-based learning. The findings also highlight the importance of teacher competence, curriculum coherence, and policy support in early algebra implementation. This study recommends strengthening concept-oriented instruction, enhancing teacher professional development, and conducting further research on the sustainability and scalability of early algebra implementation.

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INTRODUCTION

Early algebra is increasingly regarded as an essential foundation in elementary mathematics education because it emphasises students' understanding of mathematical structures and relationships from an early age. Unlike instruction that focuses solely on computational procedures, early algebra highlights students' engagement in identifying patterns, understanding equivalence, and recognising functional relationships that underlie arithmetic concepts (Kaput, 2017; Kieran, 2016). Several studies have shown that learning activities involving pattern generalisation and the use of multiple representations helped students develop a more meaningful understanding of algebra at the elementary school level (Blanton et al., 2018; Kaput & Blanton, 2005). These findings indicate that early algebra plays a crucial role in ensuring continuity in mathematics learning and in preparing students to understand more advanced algebraic concepts. Empirical evidence also shows that students' algebraic reasoning is strongly influenced by how they interpret relationships and structures rather than by procedural competence alone. A study by Fitrianna et al. (2024) demonstrated that students with stronger resilience were more likely to construct meaningful algebraic reasoning strategies, while students with lower resilience tended to rely on trial-and-error procedures and isolated computations. These findings suggest that difficulties in algebraic reasoning are closely related to students' limited engagement with relational thinking and structural understanding, which need to be cultivated systematically from earlier grades.

In contrast to traditional views that position algebra as a formal subject introduced at the secondary level, the early algebra approach emphasises that algebraic ideas such as patterns, functional relationships, equivalence, and the use of variables as quantities that can vary are introduced gradually and contextually starting in elementary school (Kaput, 2017; Kieran et al., 2016). Although various early algebra instructional approaches were reported to have positive effects on student learning, existing studies showed considerable variation in research focus, instructional design, and indicators used to examine early algebra (Blanton et al., 2018; Radford, 2012). In the elementary school context, learning obstacles further contribute to students' difficulties in developing the mathematical understanding that is essential for early algebra. Rohim et al. (2024) identified various learning obstacles experienced by elementary students, including learning disorders, teaching dysfunctions, and underachievement, which hinder students' ability to construct meaning in mathematics learning. Such obstacles indicate that without appropriate instructional support, students may experience persistent difficulties that affect their readiness to engage with early algebraic ideas. This diversity presents challenges in summarising research trends and identifying dominant and effective approaches.

Alongside developments in mathematics education research, a range of early algebra instructional approaches was developed and reported in prior studies. Research findings indicated that pattern and generalisation-based activities, the introduction of simple functional relationships, and the use of multiple mathematical representations such as diagrams, tables, and informal symbols supported elementary students in developing early algebraic understanding (Carraher & Schliemann, 2007; Mason, 2006). In addition, instruction connected to real-life contexts enabled students to relate algebraic concepts to meaningful situations, thereby increasing engagement and conceptual understanding (Jones, 2012). The use of concrete manipulatives and digital technology has also received

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increasing attention in early algebra instruction. Studies showed that interactive digital media and technology-based learning environments facilitated visual and dynamic exploration of patterns and mathematical relationships, helping students develop more flexible algebraic understanding (Mackrell & Bokhove, 2017; Sinclair & Bruce, 2015).

Despite the growing number of studies on early algebra, systematic reviews that comprehensively map research trends, dominant instructional approaches, and methodological limitations remain relatively limited. Some reviews indicated that early algebra research often focused on specific instructional interventions and their short-term effects on learning outcomes, without providing a comprehensive longitudinal perspective on the development of the field (Cai, 2011; Mulligan et al., 2013). As a result, the understanding of how algebraic ideas develop progressively and sustainably across elementary grades remains incomplete.

Furthermore, variations in educational contexts, grade levels, and the instruments and indicators used in early algebra research complicate efforts to synthesise findings. Differences in curricula, student characteristics, and assessment approaches made research results highly contextual and difficult to compare directly (Pang & Kim, 2021; Schmittau, 2005). This situation presents challenges for researchers and practitioners in identifying effective early algebra instructional practices that can be widely adapted. Therefore, a structured literature review is needed to organise existing findings, identify emerging patterns and trends, and reveal methodological limitations that require further investigation.

Based on these considerations, a structured literature study is necessary to systematically review, organise, and analyse research on early algebra in elementary education. Such a review is important not only to identify widely used and effective instructional approaches but also to reveal research gaps and limitations that warrant further attention. Accordingly, this article aims to review and synthesise research on early algebra in elementary education to obtain a systematic overview of research directions and characteristics. Specifically, this article examines: (1) the main foci and aspects addressed in early algebra research; (2) the dominant instructional approaches and methods used to develop students' algebraic thinking; and (3) existing limitations and research gaps as a basis for recommendations for future research and instructional practice. The novelty of this article lies in its integrated mapping of approaches, contexts, and research trends in early algebra, providing a conceptual framework for researchers and educators to develop more effective and sustainable early algebra instruction.

METHOD

The study employed a literature analysis method to examine research trends, instructional approaches, and challenges in early algebra research at the elementary school. The literature analysis method was selected because it allowed the researchers to critically review, compare, and synthesise findings from previously published studies in a systematic manner, thereby providing a comprehensive overview of developments within a particular field of study (Snyder, 2019; Xiao & Watson, 2017).

The literature sources were obtained from the Google Scholar database, which was chosen because it provided broad coverage of academic publications in mathematics

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education, including journal articles and conference proceedings. Google Scholar has been recognised as a relevant and reliable source for literature reviews due to its ability to index interdisciplinary publications and various types of scholarly sources (Harzing & Alakangas, 2015; Martín-martín et al., 2018).

The literature search was conducted using the keywords early algebra, early algebra in elementary schools, and early algebra in primary education. The search was limited to articles published between 2020 and 2025 to ensure the review's currency. The retrieved articles were screened based on the relevance of their titles and abstracts to the research focus. Subsequently, the full texts of the selected articles were reviewed to identify research foci, instructional approaches, and reported challenges. The analysis results were then synthesised to map research trends, dominant instructional approaches, and key challenges in the development and implementation of early algebra in elementary schools. The research procedure is presented in Figure 1.

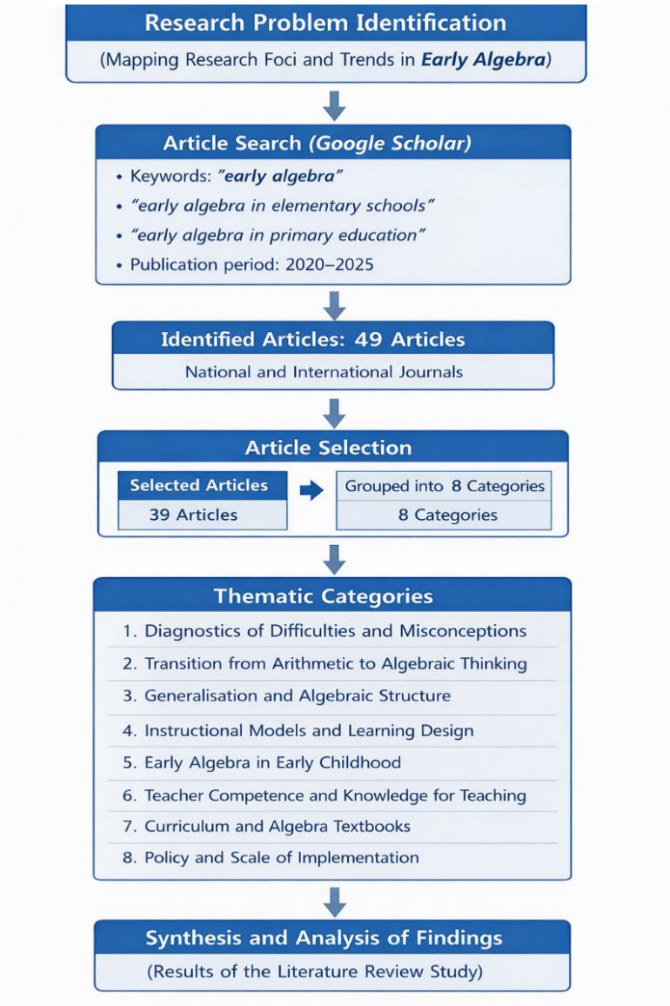

Figure 1. Research Procedure

Figure 1 illustrates the research procedure, beginning with article retrieval from Google Scholar and followed by the screening and classification of articles into eight thematic categories. Of the 49 articles identified within the 2020–2025 period, 39 articles were further analyzed to examine trends, instructional approaches, and challenges in early algebra research at the elementary school level.

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RESULTS AND DISCUSSION

The Based on the articles that have been collected, the following section presents a recap of the identified categories and the relevant articles included in each category. This summary provides a structured overview to facilitate clearer analysis and comparison across the selected studies.

Table 1. Table’s Title

No.	Category	Articles
1.	Diagnostics of Difficulties, Learning Obstacles, and Misconceptions in Early Algebra	(Engledowl, 2020; Jorgensen et al., 2004; Lim, 2022; Lumbanbatu et al., 2025; Milinković et al., 2022; Pratama et al., 2023; Simatupang et al., 2025; Sun et al., 2023; Wasitoh et al., 2023; Widyatma & Ramadhani, 2024)
2.	Transition from Arithmetic to Algebraic Thinking and Relational Reasoning	(Fauziyah & Masduki, 2023; Lim, 2022; Nada, 2023; Nemirovsky et al., 2020; Radford, 2025; Sung et al., 2021)
3.	Generalisation, Structure, and Core Activities of Algebraic Thinking	(Adamuz-Povedano et al., 2024; Fauziyah & Masduki, 2023; Kilhamn et al., 2022; Omid et al., 2022; Plessis, 2025; Strachota, 2020; Sung et al., 2021)
4.	Models, Media, and Instructional Design for Developing Early Algebra	(Adamuz-Povedano et al., 2024; Alsina et al., 2024; Chynthia et al., 2024; Permatasari et al., 2021; Prasetyo & Indianasari, 2024; Sylviani & Permana, 2020; Tomohiro et al., 2021)
5.	Early Algebra in Early Childhood and Early Elementary Grades	(Ariyana, 2022; Calor et al., 2020; Mulligan et al., 2013)
6.	Teacher Competence and Knowledge for Teaching Early Algebra	(Aydoğan & Büyükşahin, 2023; Eriksson, 2022; Pincheira & Alsina, 2021, 2024; Plessis, 2025; Rusani et al., 2025)
7.	Curriculum, Textbooks, and Conceptual Representations of Algebra	(Do, 2022; Draper & Lott, 2020; Helenius & Ahl, 2024)
8.	Policy, Scale of Implementation, and Social Context of Early Algebra	(Adamuz-Povedano et al., 2024; Brummet et al., 2023)

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Diagnostics of Difficulties, Learning Obstacles, and Misconceptions in Early Algebra
The articles in this category provided a detailed portrayal of the nature of early algebra difficulties experienced by students through diagnostic approaches, including error analysis, interviews, and written test instruments. The findings consistently indicated that these difficulties were not singular in nature but instead resulted from an interconnected combination of conceptual, procedural, and representational problems (Lumbanbatu et al., 2025; Milinković et al., 2022; Pratama et al., 2023; Simatupang et al., 2025; Wasitoh et al., 2023; Widyatma & Ramadhani, 2024).

From a conceptual perspective, misconceptions about variables emerged as the most dominant issue. The studies reported that students tended to interpret variables as specific values that must be immediately determined, rather than as symbols representing general quantities or relationships among quantities (Lumbanbatu et al., 2025; Milinković et al., 2022; Simatupang et al., 2025). This interpretation was evident in students' responses, where they attempted to "guess" variable values through random numerical substitution or by forcing familiar arithmetic operations. As a result, students struggled to understand algebraic expressions as representations of relationships and instead perceived them as hidden arithmetic calculations. This pattern aligns with existing views that early algebra requires a shift from value-oriented to relational thinking.

Another prominent conceptual misconception concerned the meaning of the equal sign ("="). The articles showed that most students interpreted the equal sign operationally, as an indicator of the final result of a calculation, rather than as a symbol expressing the equivalence of two mathematical expressions (Pratama et al., 2023; Simatupang et al., 2025; Sun et al., 2023). This misconception was reflected in students' failure to solve non-canonical or open equations, as they expected a single expression on the left and a single result on the right. More systematic assessment studies demonstrated that this misconception appeared consistently across grade levels and problem contexts (Engledowl, 2020; Jorgensen et al., 2004), indicating its persistent nature if left unaddressed.

From a procedural standpoint, the articles revealed a strong reliance on direct calculation and trial-and-error strategies (Simatupang et al., 2025; Wasitoh et al., 2023). While these strategies were effective in arithmetic contexts, they became sources of error when students encountered algebraic tasks requiring generalisation or modelling. Procedural errors, such as meaningless symbol manipulation or the use of irrelevant operations, reflected weak connections between procedures and the underlying concept.

Representational difficulties also emerged as a major source of error. The studies reported that students experienced significant obstacles in translating contextual situations or word problems into algebraic models (Lim, 2022; Milinković et al., 2022; Sun et al., 2023). This failure was not primarily caused by reading difficulties, but rather by students' inability to identify relevant quantities, relationships, and mathematical structures. Research examining number sense and basic operations indicated that weak foundations in number and fraction concepts further exacerbated representational difficulties in early algebra (Lumbanbatu et al., 2025; Widyatma & Ramadhani, 2024).

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Several articles employed learning obstacle frameworks to explain the roots of these difficulties (Pratama et al., 2023; Wasitoh et al., 2023). Ontogenic obstacles were associated with students' prior learning experiences that heavily emphasised procedural arithmetic, epistemological obstacles stemmed from the abstract and relational nature of algebraic concepts, and didactical obstacles arose from instructional designs that provided limited opportunities to explore the meaning of symbols and relationships. These obstacles did not operate independently; instead, they reinforced one another. Overall, the articles in this category confirmed that early algebra difficulties and misconceptions resulted from a complex interaction between the nature of algebraic concepts, students' prior arithmetic learning experiences, and instructional practices that continue to emphasise procedures over relational understanding.

Transition from Arithmetic to Algebraic Thinking and Relational Reasoning

The articles in this category comprehensively highlighted the transition from arithmetic to algebraic thinking as a crucial yet problematic phase of cognitive development. The review findings showed that although students had been introduced to algebraic symbols and notation, their modes of thinking remained dominated by arithmetic approaches based on direct calculation and numerical manipulation (Lim, 2022; Nada, 2023; Radford, 2025). These findings indicated that the introduction of algebraic symbols alone did not automatically transform how students understood and solved mathematical problems.

One major difficulty identified was students' weak relational reasoning. Several studies reported that students struggled to understand relationships among quantities, particularly in situations involving covariation or simultaneous changes between variables (Nemirovsky et al., 2020; Radford, 2025; Sung et al., 2021). When one quantity changed, students tended to focus on visible numerical values without considering the structural relationships linking them to other quantities. As a result, the predictions students made were often partial, inconsistent, and highly dependent on specific examples rather than on general relationships.

In problem-solving contexts, the articles revealed the dominance of value substitution and trial-and-error strategies as characteristic of the arithmetic to algebra transition phase (Fauziyah & Masduki, 2023; Nada, 2023; Nemirovsky et al., 2020). Students attempted to "convert" algebraic problems into more familiar arithmetic forms by inserting particular values or performing direct calculations. This strategy indicated that students did not yet view algebraic expressions and equations as representations of general relationships, but rather as incomplete calculations awaiting numerical completion.

The articles also emphasized the important role of representations in supporting the transition to algebraic thinking. Instructional approaches involving multiple representations, such as visual, verbal, and symbolic, showed potential for helping students observe regularities and understand quantitative change (Nemirovsky et al., 2020; Sun et al., 2023; Sung et al., 2021). Visual representations such as tables, diagrams, and graphs enabled students to perceive patterns and relationships that were not immediately apparent through numerical calculations. Classroom discussions play a key role in shifting students' focus from computational results to the underlying mathematical relationships.

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However, the effectiveness of these approaches depends heavily on the quality of teacher mediation. The articles reported that without explicit guidance, students often failed to connect different representations coherently (Fauziyah & Masduki, 2023; Nemirovsky et al., 2020). Inconsistencies across representations remain a major challenge, as students may understand a situation in one form but are unable to transfer that understanding to another, particularly to formal symbolic representations. Another prominent challenge identified was the dominance of everyday narrative language over relational mathematical language. Students were able to explain problem situations verbally using informal language but struggled to express the same relationships in symbols, equations, or formal relational forms (Fauziyah & Masduki, 2023; Nemirovsky et al., 2020). Overall, the articles in this category confirmed that the transition from arithmetic to algebraic thinking was not a merely technical shift, but a fundamental change in ways of reasoning that required carefully designed learning experiences. This transition requires instruction that emphasises relationships, structure, and connections.

Generalisation, Structure, and Core Activities of Algebraic Thinking

The articles in this category consistently positioned generalization as the core of algebraic thinking and as a primary indicator of students' development of algebraic reasoning. Generalisation was understood not merely as the ability to find general rules, but as a cognitive process involving the recognition of structures, relationships, and regularities that extend beyond specific concrete examples (Plessis, 2025; Strachota, 2020). Accordingly, students' ability to generalise patterns and relationships served as an important measure of the extent to which they had progressed from arithmetic thinking toward algebraic thinking.

However, the review findings showed that students' early generalisations were often intuitive and context-bound, and heavily dependent on the examples provided (Fauziyah & Masduki, 2023; Sung et al., 2021). Students were able to recognise regularities in numerical sequences or specific visual patterns, but they struggled to express these regularities in a general form or to connect them to other situations sharing the same structure. These findings indicated that many students' generalisations remained at an empirical level and had not yet reached the level of structural generalisation that characterises algebraic thinking.

Several articles highlighted a fundamental distinction between recognising surface-level patterns and understanding the underlying mathematical structures (Kilhamn et al., 2022; Omid et al., 2022). Students often succeeded in extending patterns or identifying rules based on direct observation, but they failed to explain the mathematical reasons underlying those patterns. When the context or representation of a pattern was changed, generalisations that had previously appeared stable became inconsistent. This pattern suggests that students had not yet fully grasped the relational structures underlying the patterns, but instead relied on visible surface features.

The instructional approaches examined in this category emphasised the importance of tasks based on patterns, functions, and relationships that were explicitly designed to promote generalisation (Adamuz-Povedano et al., 2024; Strachota, 2020). These tasks not only ask students to identify patterns, but also require them to justify their reasoning, predict general cases, and test the validity of their generalisations across multiple examples. The articles demonstrated that tasks involving higher cognitive demand

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produced more meaningful generalisations than tasks that merely required repetition or simple pattern extension.

The findings of Plessis (2025) indicated that classroom discussion and mathematical justification emerged as essential components in the development of stable generalisations. Through discussion, students were encouraged to articulate their thinking, compare different strategies, and revise inadequate generalisations. This process helped students move generalisations toward structured ones. However, the articles also noted that without targeted teacher guidance, classroom discuss could remain at a descriptive level without reaching structural understanding.

Social interaction, particularly through interactive group work, enriched the generalisation process by bringing together diverse perspectives and ways of thinking (Adamuz-Povedano et al., 2024). In these interactions, students did not merely compare answers, but also discussed the reasoning and structures underlying their generalisations. Nevertheless, the articles emphasised that the quality of task design and teacher questioning was the primary factor determining the depth of generalisation achieved. Tasks that were insufficiently challenging or questions that were overly directive tended to result in shallow generalisations that were not robust across contexts. Overall, this category confirmed that the development of algebraic thinking through generalisation requires instruction that focuses on structure, process, and justification. Meaningful generalisation does not emerge spontaneously, but rather results from the interaction of well-designed tasks, classroom discuss, and teachers' facilitation of students' ways of thinking.

Models, Media, and Instructional Design for Developing Early Algebra

The articles in this category highlighted the strategic role of instructional models, learning media, and task design in supporting the development of students' algebraic thinking from an early stage. The primary focus of these studies was not limited to learning outcomes, but also examined how students' modeling, reasoning, and reflection processes were facilitated through innovative instructional designs (Adamuz-Povedano et al., 2024; Alsina et al., 2024; Chynthia et al., 2024; Permatasari et al., 2021; Prasetyo & Indianasari, 2024; Sylviani & Permana, 2020; Tomohiro et al., 2021).

Problem-Based Learning (PBL) emerged as one of the approaches examined in the context of early algebra (Adamuz-Povedano et al., 2024; Chynthia et al., 2024). The articles showed that PBL encouraged students to engage with open-ended problem situations that required relational modelling and the search for underlying structures, rather than the mere application of procedures. Through processes of problem exploration, discussion, and reflection, students began to develop an understanding of variables, relationships, and equivalence. However, the findings also indicated that without adequate scaffolding, PBL risked becoming contextual problem-solving activities that were disconnected from the intended algebraic concepts.

Game-based learning approaches, particularly through the use of digital media such as DragonBox Algebra, demonstrated strong potential in building students' understanding of algebraic principles (Sylviani & Permana, 2020). The study reported that game environments allowed students to interact with concepts of equivalence and algebraic

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transformation without the pressure of formal symbolic notation. Students showed increased algebraic intuition, especially in understanding operations as transformations that preserve equivalence. Nevertheless, the article also noted that the transfer from game-based understanding to formal symbolic representations did not occur automatically and required further pedagogical intervention. Several articles emphasised the use of visual scaffolding and self-explanation strategies as means to enhance students' metacognitive awareness in algebraic thinking (Permatasari et al., 2021; Tomohiro et al., 2021). Visual representations such as diagrams, concrete models, and animations helped students make sense of relationships and structures. At the same time, these representations had the potential to impose cognitive overload if their number or complexity was not aligned with students' prior knowledge.

The APOS (Action, Process, Object, Schema) approach was examined as a theoretical framework for constructing algebraic concepts in a gradual and structured manner (Prasetyo & Indianasari, 2024). The findings indicated that APOS was effective in supporting students' transition from concrete actions to more abstract algebraic objects. However, the success of this approach depended heavily on teachers' readiness to design learning activities that were aligned with students' cognitive stages. The main challenges reported in this category included students' low initial levels of algebraic thinking, variations in teachers' competencies, and the dependence of digital media on adequate technological infrastructure (Alsina et al., 2024). Collectively, these articles confirm that innovations in instructional models and learning media cannot stand alone, but must be supported by careful pedagogical planning, sustained teacher professional development, and realistic implementation contexts.

Early Algebra in Early Childhood and the Lower Grades of Elementary School

The articles in this category affirmed that the development of algebraic thinking does not need to wait until the introduction of formal symbols, but can begin in early childhood and the lower grades of elementary school. The focus of these studies was not on symbolic manipulation, but on the development of patterns, relationships, equivalence, and non-symbolic generalisation that are appropriate to children's cognitive developmental stages (Ariyana, 2022; Calor et al., 2020).

The findings of Ariyana (2022), showed that through play-based learning and concrete exploratory activities, children were able to recognise regularities, compare quantities, and make simple predictions based on observed patterns. Play activities involving concrete objects, repeated games, and everyday contextual situations enabled children to build relational understanding intuitively. In this context, generalisation was not expressed through symbols or formulas, but rather through natural language, gestures, and actions, which reflect early forms of algebraic thinking.

The results of Calor et al. (2020) reinforced these findings by demonstrating that children were able to develop non-symbolic generalisations when instruction followed clear and gradual learning trajectories. Children could understand the concept of equivalence as a relationship of "the same amount" or "balance" through object manipulation and concrete representations before being introduced to the "=" symbol. Sociocultural scaffolding, such as open-ended questions and dialogic interactions between teachers and children, played an important role in helping children reflect on the relationships and regularities they discovered.

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Despite the clear potential of early algebra at the early childhood level, Mulligan et al. (2013) identified several implementation barriers. The primary obstacle was the perception of algebra as abstract and “too difficult” for young children. This perception influenced instructional practices that emphasised simple counting and mastery of numerical facts, thereby limiting opportunities to develop patterns and relationships. Teachers tended to avoid activities involving relational reasoning because they were perceived as inappropriate for children’s abilities. In addition, the article showed that the interpretation of the equal sign at early grade levels remained operational. Children were often introduced to the “=” symbol merely as an indicator of a final result, rather than as a symbol of equivalence between two quantities. Such instructional patterns risk establishing persistent misconceptions that may affect algebra learning at later stages.

Overall, the articles in this category confirmed that early algebra in early childhood and the lower elementary grades can be effectively developed through pedagogically appropriate approaches that align with children’s cognitive development. The main challenges do not lie in children’s cognitive readiness, but rather in teachers’ understanding, perceptions of algebra, and the continued dominance of instructional practices oriented toward procedural counting. These findings strengthen the argument that early algebra should be viewed as a way of thinking cultivated gradually from an early age, rather than as a symbolic subject matter.

Teacher Competence and Knowledge for Teaching Early Algebra

The articles in this category consistently emphasised that teacher competence, particularly Mathematical Knowledge for Teaching (MKT), was a key factor determining the success of early algebra development in classrooms. The focus of these studies was not limited to teachers’ general mastery of mathematical content, but rather centred on their ability to understand algebraic structures, anticipate students’ ways of thinking, and design instructional interactions that promote generalisation and relational reasoning (Aydoğan & Büyükşahin, 2023; Pincheira & Alsina, 2021, 2024; Plessis, 2025).

One major finding that emerged across studies was the weakness of teachers’ Specialised Content Knowledge (SCK) in the context of early algebra (Eriksson, 2022; Pincheira & Alsina, 2024). The articles showed that many teachers were able to solve algebraic problems procedurally, but struggled to explain the meanings underlying symbols, relationships, and structures. This limitation had a direct impact on instructional practices, as teachers tended to emphasise procedures or specific examples without supporting students in understanding the generalisations behind them. Consequently, opportunities to develop students’ conceptual algebraic thinking were limited.

In addition to SCK, weak horizon content knowledge was identified as another important issue (Pincheira & Alsina, 2021; Plessis, 2025). Teachers often lacked a comprehensive view of algebraic learning trajectories across grade levels, which made it difficult for them to connect early algebra activities with formal algebra learning at later stages. The articles indicated that without sufficient horizon knowledge, teachers tended to view algebra as a separate symbolic topic rather than as a way of thinking that should be developed progressively from an early age.

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Eriksson (2022) & Rusani et al. (2025) specifically highlighted teachers' difficulties in anticipating common student misconceptions, such as interpreting variables as fixed values or the equal sign as indicating a result. Teachers who lacked a deep understanding of typical student error patterns tended to respond reactively, for example, by providing correct answers or procedures, rather than using these misconceptions as starting points for classroom discussion. This pattern suggests that the ability to diagnose and respond to students' thinking is a critical component of MKT in early algebra instruction.

Several articles also examined the effectiveness of professional development interventions, such as workshops and reflective teaching practice programs (Aydoğan & Büyükşahin, 2023; Rusani et al., 2025). The findings indicated that these activities improved teachers' awareness of the importance of structure and generalisation in algebra and enriched their pedagogical strategies. However, the gains in competence tended to be temporary if they were not supported by sustained mentoring and professional learning communities. Overall, this category confirmed that the development of early algebra depends heavily on the quality of teachers' instructional knowledge. Limitations in MKT, particularly in SCK and horizon content knowledge, are not merely individual teacher issues, but rather reflect broader shortcomings in professional development systems that have not yet fully positioned early algebra as a strategic instructional priority.

Curriculum, Textbooks, and Conceptual Representations of Algebra

The articles in this category examined the role of curricula and textbooks in shaping students' conceptual understanding of algebra, particularly during the early stages of algebraic thinking development. The findings revealed conceptual and pedagogical discontinuities in the presentation of algebraic ideas, indicating that early algebra was not positioned as a foundational basis for formal algebra learning (Do, 2022; Draper & Lott, 2020; Helenius & Ahl, 2024).

The study by Helenius & Ahl (2024) found that in many curricula and textbooks, algebraic concepts were introduced abruptly at certain grade levels in the form of symbols and formal procedures. Before this stage, mathematics instruction focused primarily on arithmetic, with limited emphasis on relationships, structure, and generalisation. This gap resulted in students facing a sharp conceptual leap when entering algebra instruction, without having developed sufficient algebraic ways of thinking.

The dominance of symbolic algebra emerged as another key finding (Do, 2022; Draper & Lott, 2020). Textbook content analyses showed that algebraic representations were largely presented through symbolic equations and formal manipulations, while relational contexts and the conceptual meanings of symbols received little attention. This pattern reinforces the view of algebra as a collection of rules and procedures rather than as a way of thinking centred on relationships among quantities.

One of the most problematic representational aspects was the interpretation of the equal sign ("="). The articles indicated that textbooks frequently presented the equal sign in an operational context, namely as a marker of the final result of a calculation (Draper & Lott, 2020; Helenius & Ahl, 2024). This presentation contributed to the development of persistent operational misconceptions, in which the equal sign was not understood as representing the equivalence of two expressions. Such misconceptions carry over to

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higher grade levels and become significant obstacles to understanding relational algebraic equations.

In addition, Do (2022) highlighted the limited cross-grade coherence in the presentation of algebraic concepts. Although some early algebra elements, such as patterns and relationships, appeared in the early grades, they were not developed further nor explicitly connected to algebra learning at later stages. Consequently, the potential of early algebra as a conceptual bridge was underutilised. Overall, the articles in this category confirmed that curricula and textbooks play a central role in shaping students' views of algebra. Conceptual gaps across grade levels, symbolic dominance, and repeated operational representations underscore the need for curriculum and textbook redesign so that early algebra can be systematically integrated into students' understanding of algebraic structure.

Policy, Scale of Implementation, and the Social Context of Early Algebra

The articles in this category framed early algebra not merely as a classroom-level pedagogical innovation, but as a systemic issue that was strongly shaped by education policy, social context, and large-scale implementation challenges. The analytical focus shifted from asking whether early algebra is effective to examining under what conditions and for whom early algebra can be implemented sustainably (Adamuz-Povedano et al., 2024; Brummet et al., 2023).

Brummet et al. (2023) investigated the impact of early algebra policy initiatives using quantitative policy analysis approaches, such as Regression Discontinuity Design (RDD). The findings indicated that policies promoting early algebra produced positive effects on certain learning outcomes; however, these effects were not uniform across schools or student groups. Variations in school contexts, student composition, and implementation quality resulted in differential policy outcomes. These results demonstrate that the success of early algebra depends not only on instructional design, but also on contextual conditions at the school and system levels.

A central issue highlighted in this category was the challenge of scaling, namely the difficulty of replicating successful early algebra programs from limited contexts to broader system-wide implementation (Brummet et al., 2023). The study showed that interventions that were effective on a small scale often lost their effectiveness when implemented at scale, particularly in the absence of sufficient resources, sustained teacher professional development, and robust evaluation mechanisms. Resource inequities emerged as a critical factor influencing implementation quality and risked exacerbating existing educational inequalities.

Adamuz-Povedano et al. (2024) added a social-context dimension by examining problem-based learning approaches in early algebra that incorporated social dilemmas or algebraic wicked problems. In this approach, algebra was positioned as a tool for understanding and reflecting on complex issues involving values, sustainability, and social responsibility. The findings suggested that such approaches enhanced the relevance of mathematics for students and expanded the meaning of learning algebra beyond symbolic manipulation. However, the study also highlighted the dual complexity that arises when mathematical learning is integrated with social contexts. Students were required not only to understand

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algebraic structures, but also to interpret contextual information, values, and the social implications of their solutions. This situation demands high levels of pedagogical readiness from teachers and instructional designs that are sensitive to students' social and cultural backgrounds.

Both articles consistently emphasised the challenge of assessing non-traditional learning outcomes. Early algebra instruction that focuses on reasoning, modelling, and social reflection often produces learning outcomes that are not easily captured by standardised tests (Adamuz-Povedano et al., 2024; Brummet et al., 2023). Overall, this category underscores that the large-scale success of early algebra relies on alignment among policy frameworks, social contexts, teacher capacity, and assessment systems. Early algebra cannot be treated as a purely technical intervention, but rather must be understood as part of a broader educational system transformation that requires sustained attention and concrete implementation.

CONCLUSION

This article reviews and synthesises research on early algebra in elementary schools to provide a systematic overview of the directions and characteristics of existing studies. The review shows that early algebra research has developed rapidly and covers a wide range of foci, including analyses of students' difficulties and misconceptions, the transition from arithmetic to algebraic thinking, and the development of generalisation, instructional models, and education policies. These findings confirm that early algebra is not merely about the introduction of symbols, but constitutes a form of mathematical thinking that emphasises relationships, structure, and generalisation.

In terms of research focus, most studies highlight students' conceptual difficulties, particularly in understanding variables, the equal sign, and relationships among quantities. In addition, many studies indicate that students continue to rely on arithmetic strategies even after being introduced to algebraic forms. Research on generalisation and structure shows that students can recognise patterns, but often struggle to understand the underlying mathematical reasoning behind those patterns. Studies involving early childhood and lower elementary grades also demonstrate that algebraic thinking can be developed from an early age through non-symbolic approaches, although classroom practice remains largely dominated by basic computational instruction.

From the perspective of instructional approaches, the reviewed studies suggest a shift toward the use of multiple representations, pattern and relation-based tasks, classroom discussion, and innovative instructional models such as problem-based learning and game-based learning. These approaches have the potential to support more meaningful understanding of algebra, but their effectiveness depends strongly on the quality of task design and the teacher's role in mediating learning. The findings also emphasise that teacher competence, particularly mathematical knowledge for teaching, is a key factor in the successful development of early algebra.

Nevertheless, this review also reveals several limitations and research gaps. Longitudinal studies that trace the development of students' algebraic thinking across grade levels are still limited, as are studies that examine the roles of curriculum, textbooks, and policy in depth. In addition, the assessment of early algebra learning outcomes continues to rely

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heavily on traditional tests that are not well-suited to capturing students' relational reasoning. Therefore, future research and instructional practice will need to integrate early algebra more systemically by addressing teacher readiness, curriculum coherence, and the development of assessment approaches that will align with the nature of algebraic thinking.

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